Name Khaled Jedollah, ID 38264

Exam I , MTH 221 , Spring 2011

Ayman Badawi

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QUESTION 1. (39 POINTS) For each question below, circle the right answer.

(i) Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 7 \end{bmatrix}$. Then A^{-1}

a. Does not exist

b. $\begin{bmatrix} 1 & -2 \\ 4 & -7 \end{bmatrix}$.

c. $\begin{bmatrix} 7 & 4 \\ -2 & -1 \end{bmatrix}$.

d. (a) and (b) and (c) are wrong

(ii) Let $A = \begin{bmatrix} -4 & 2 & 3 & b \\ 4 & -4 & d & a \\ 8 & -4 & -5 & c \\ 12 & -6 & -9 & -3b+2 \end{bmatrix}$. Then $det(A)$

a. It depends on the values of a, b, c, d

b. it is always -90

c. it is always 16

d. it is always -90(-3b+2)

(iii) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -4 & 1 \end{bmatrix}$. Then $A^{-1} =$

a. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0.5 & -0.25 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix}$.

c. $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

d. None of the above

(iv) Let A, B be 3×3 matrices such that $2AB = I_3$ and $det(A) = 2$. Then $det(B^{-1}) =$

a. 16

b. 0.5

c. 4

d. None of the above.

(v) Let A, B as in the previous question. One of the following is a possibility for A^{-1}

a. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 0.5 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 4 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 4 & 2 \end{bmatrix}$

d. $\begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(vi) Let A, B as in question (IV). Then $det(2BA - 3I_3)$

a. -2

b. -8

c. -26

d. Can NOT BE DECIDED (i.e. More information is needed)

(vii) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}$, and $C = \begin{bmatrix} 2a_1 & 2a_2 + a_3 & 2a_3 \\ 2a_4 & 2a_5 + a_6 & 2a_6 \end{bmatrix}$. Given that B is a 3×3 matrix such that $AB = C$. Then $B =$

a. $\begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

d. None of the above

(viii) Let A be a 3×3 matrix such that $A \xrightarrow{-2R_1 + R_3 \to R_3} A_1 \xrightarrow{3R_2 + R_3 \to R_3} A_2 \xrightarrow{2R_1} B = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$. Then $det(A) =$

a. -1

b. 4

c. 8

d. 2

(ix) Consider Question (VIII). Let F be a matrix such that $FB = A$. Then $F =$

a. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 3 & 1 \end{bmatrix}$

b. $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 3 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 3 & 2 \end{bmatrix}$

+3

No Answe

$F = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix}$ Should Be

(x) in Question (VIII). One of the following statements is correct:

a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A_1 = B$

b. $\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} A_1 = B$

c. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} A_1 = B$

d. NONE OF THE ABOVE

(xi) Given A is a 2×2 matrix such that $\left(A \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1} = 0.5 I_2$. Then $A =$

a. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$

c. $\begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}$

d. NONE OF THE ABOVE

(xii) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ and $F = \begin{bmatrix} 2a_1 + a_3 \\ 2a_4 + a_6 \\ 2a_7 + a_9 \end{bmatrix}$. Given $det(A) = 0$. Then the system of linear equations $AX = F$:

a. has infinitely many solutions and $x_1 = 2, x_2 = 0, x_3 = 1$ is a solution to the system.

b. has unique solution $x_1 = x_3 = 1$ and $x_2 = 0$

c. has infinitely solutions and $x_1 = x_3 = 1, x_2 = 0$ is a solution.

d. NONE OF THE ABOVE.

(xiii) Let $C = \begin{bmatrix} 0 & 1 & 4 \\ 1 & -1 & 3 \\ -1 & -1 & a \end{bmatrix}$. The system of linear equations $CX = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ is INCONSISTENT if

a. $a = -11$
b. $a = -4$
c. $a = 4$
d. $a = -7$

QUESTION 2. (11 POINTS) Solve the following system:

$x_2 - x_3 + x_5 = 0$
$x_1 + x_3 - x_4 = 2$
$x_2 + x_3 + x_4 + x_5 = 4.$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 2 \\ 0 & -1 & 1 & 1 & 1 & 4 \end{array}\right] \quad R_1 + R_3 \longrightarrow R_3$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{array}\right] \quad R_3 + R_2 \longrightarrow R_2$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 & 6 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{array}\right]$$

Read: x_1, x_2, x_4 are leading vari
x_3, x_5 are free variables

$x_2 - x_3 + x_5 = 0$
$x_1 + x_3 + 2x_5 = 6$
$x_4 + 2x_5 = 4$

eral Solution:

$$\boxed{\begin{aligned} x_1 &= 6 - x_3 - 2x_3 \\ x_2 &= x_3 - x_5 \\ x_4 &= 4 - 2x_5 \end{aligned}}$$

one solution: $x_1 = 6$
$x_4 = 4$
$x_2 = x_3 = x_5 = 0$